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Transport of Quantum Correlations across a spin chain

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Some of the recent developments concerning the propagation of quantum correlations across spin channels are reviewed. In particular, we focus on the improvement of the transport efficiency obtained by the manipulation of few energy parameters (either end-bond strengths or local magnetic fields) near the sending and receiving sites. We give a physically insightful description of various such schemes and discuss the transfer of both entanglement and of quantum discord.

Keywords: spin models; entanglement; quantum communication.

1. Introduction

The problem of entanglement distribution has become of central interest in quantum information theory and quantum communication: quantum correlations are generated by local interactions; therefore methods are required to transfer either the entangled particles or their state at a distance. It has been shown theoretically that spin chains can behave as efficient quantum channels for short distance entanglement transfer^{1,2}, and that, for such cases, the transfer of entanglement is strictly linked to the transport of a quantum bit of information along the chain. As a consequence, it has become more and more clear that the ability to manipulate the properties of a spin chain, intended as a qubit register, can be very important for generic communication purposes.

A simple protocol for information transmission can work as follows: the qubit state to be transmitted is prepared at one end of the chain, on the spin residing at the first site, and is then propagated to the other end due to the time evolution generated by the spin Hamiltonian, which might be time-dependent if additional external controls are included (see, e.g. the proposals put forward in Ref.³). Even if, for short-length chains, the fidelity of state transfer is close to unity, it inevitably degrades with increasing the communication distance: since the initial state is localized, many spin excitations are typically involved in the dynamics, causing the dispersion of the initial information over the entire length of the chain. Thus, as this length increases, the fidelity of state transfer substantially reduces and various kinds of quantum correlations are generated which spreads all over the chain length.

To overcome this problem, several schemes have been proposed to achieve perfect state transfer, e.g. by encoding the information in low-dispersion gaussian wave packet⁴, by using local memories⁵, or by “conclusive” transfer, in which parallel quantum channels are used, supplemented by a measurement at the receiving end, allowing a high fidelity state transfer, more robust with respect to decoherence and to non-optimal timing than the single chain scheme⁶.

Another possibility is to produce a refocusing of the information at the receiving site. This can be achieved by breaking the translational invariance of the chain,^{7,8,9,10}. In particular, one can obtain perfect quantum transmission if the chain has properly engineered coupling strengths and/or local fields⁷, carefully designed in order to obtain the so called parity matching condition¹¹ in the dispersion relation of spin excitations, which guarantees a fully constructive interference at the receiving site in the presence of mirror inversion.

In general, indeed, the dispersion relation is non-linear; but one can select almost-linear regions (and, therefore, get an approximate fulfillment of the above condition) to obtain a very high transmission fidelity by letting the state to be sent to contain only excitations lying in this linear region¹².

Furthermore, it has been shown that, for such systems, one can even relax the need for chain initialization in a reference (fiducial) state⁹, provided end-chain single qubit operations can be performed. Thus, the control over the core part of the spin medium is relaxed in favor of controllability of the first and last element of the chain.

State transfer has been studied, both theoretically and experimentally, with liquid-¹³ and with solid-state NMR¹⁴ and polar molecules¹⁵, for small and larger number of spins, respectively. In such cases, one typically has access to many chain parameters, which can be even controlled in time in order to achieve perfect transmission.

Another promising implementation of information transfer protocols is obtained with trapped ions, which have been recently used to simulate the dynamics of various spin systems¹⁶.

Different experimental implementations are expected to permit a more restricted access to the effective Hamiltonian parameters¹⁷, and therefore other methods and protocols for a high fidelity transmission have been proposed, which rely on less demanding control requirement¹⁸. One possibility is to weaken the links between the end-spins (sender and receiver) and the rest of the spin chain^{19,20}, or to rely on local magnetic field control²¹; these schemes, however, both have the drawback of increasing the transmission time, implying a longer exposition of the channel itself to unwanted decoherence.

Apart from NMR-related schemes, much of the work described above has been done with one-dimensional spin systems with short range interactions, but chains with long-range interactions have also been addressed^{22,23}, and more general networks have been considered²⁴, as well as the possibility to store information²⁵, rather than transmit it.

The propagation of quantum information along the chain has been also analyzed in the presence of disorder and Anderson localization within the spin channel²⁶, or in presence of local environments coupled to the spins, which can limit the distance over which quantum information can be transmitted along the chain²⁷; moreover, environmental correlations can play a role in such cases²⁸.

The transfer of entanglement (and, more generally, of quantum correlations,²⁹) along the chain, is strictly related to the qubit state transfer described above. The generic setting is sketched in Fig. 1, where the first spin of the chain is initially prepared in an entangled state with an external, uncoupled qubit. The aim, in this case, is to obtain, at the end of the protocol, the greatest possible amount of entanglement between this external qubit and the spin residing at the last site of the chain.

In this paper we will concentrate precisely on such a process, by reviewing various possible strategies to achieve a high quality transmission of entanglement, based on static Hamiltonian which include the least possible modification of the energy parameters with respect to the case of a uniform chain, originally treated in Ref. ¹. In fact, as mentioned above, if all of the Hamiltonian parameters (meaning all of the coupling strengths between the spins and the effective local magnetic fields) are assumed to be accurately controllable, and set to specific optimal values, perfect entanglement transfer can be achieved. The accuracy with which the parameters need to be fixed, however, increases with the size of the chain, and the process of engineering the transport properties of the chain quickly becomes very demanding. It is possible, however, to achieve a very good transfer fidelity with low-complexity schemes in which only few energy parameters are assumed to be controllable from outside (for a generalization of this concept, see Ref. ³⁰, where the general case of an on-demand transfer between any pair of selected sites is treated). Specifically, we will focus on the cases in which either two local fields (that is, the energy splitting of the qubits at the sending and receiving sites) or two bond strengths (the first and the last ones) are modified with respect to the rest of the chain. We will see that, for a chain of any given size, these parameters can be optimized in order to increase the amount of transferred entanglement, still maintaining a reasonably short transfer time.

Much of this review is dedicated to the transport of entanglement, but we will also briefly discuss the propagation of general quantum correlations, as quantified by the quantum discord, which, when the system is far from its optimal transport conditions, can be favored with respect to that of entanglement.

The remainder of this paper is organized as follows: we will first introduce the Hamiltonian model that describes a generic one-dimensional spin system with nearest neighbor interactions in Sect. 2. Then, in Sec. 3, we will concentrate on a specific case, the so called *XX* model, whose performance will be then compared with that of other anisotropic models in Sec. 4. Finally, some concluding remarks are given in Sec. 5.

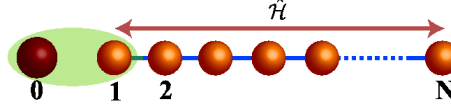


Fig. 1. Sketch of a spin channel with N particles residing at the sites of a one-dimensional lattice (an open-ended chain), whose dynamics is governed by the Hamiltonian $\hat{\mathcal{H}}$. An external, uncoupled qubit (spin 0) and the first spin of the chain share some quantum correlations to be transferred towards the other end of the chain. The pair is initially prepared in the state $\rho^{(01)}(0)$.

2. General Hamiltonian model for the spin channel

Very different strategies have been proposed in order to improve the transfer capability of quantum correlations across a spin chain. In this review, we consider only static couplings and deal with quantum channels modelled by nearest-neighbor Heisenberg-like Hamiltonians in one dimension³¹, that describe short ranged and anisotropic exchange interaction between spins $\frac{1}{2}$ particles sitting on a line, and subjected to effective local magnetic fields h_i :

$$\hat{\mathcal{H}} = \sum_{i=1}^{N-1} J_i \left((1+\gamma) \hat{S}_i^x \hat{S}_{i+1}^x + (1-\gamma) \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z \right) - \sum_{i=0,1} h_i \hat{\sigma}_i^z \quad (1)$$

where \hat{S}_i^α ($\alpha = x, y, z$) are the spin operators of the particle residing at site i of an open-ended chain, with $i = 1, \dots, N$; J_i is the interaction strength between neighbor spins; γ and Δ are the degree of anisotropy of the coupling in the XY -plane and along the Z -axes, respectively, while h is the magnetic field. As usual, the eigenstates of S_i^z are taken as the logical basis for qubit i , $\{|0\rangle_i, |1\rangle_i\}$.

This general Hamiltonian is able to describe very different physical settings and encompasses many different features depending on the values of the anisotropy parameters γ and Δ . We describe various cases in the following, but mostly concentrate on the so called XX -model, corresponding to $\gamma=\Delta=0$, while leaving a brief description of the transmission of quantum correlation in spin chains with anisotropic couplings to the last section. There, we review the comparison among the performances of various models, originally performed in Ref. ³², where it is shown that the case of a spin chain with an isotropic coupling in the equatorial plane and with transverse field is, in fact, the most favorable.

3. Transport of correlations in the isotropic XX model

Let us first consider the simplest case, known as XX model, with $\gamma=\Delta=0$.

The XX Hamiltonian can be mapped, via the Jordan-Wigner transformation, into a spinless fermion hamiltonian, up to an irrelevant constant:

$$\hat{\mathcal{H}}_{xx} = \sum_{i=1}^{N-1} \frac{J_i}{2} (c_{i+1}^\dagger c_i + h.c.) - \sum_{i=1}^N h_i c_i^\dagger c_i \quad (2)$$

which is equivalent to a tight-binding hopping model in which the fermion annihilation operator at site i is related to the spin operators as

$$c_i = \left(\prod_{j=1}^{i-1} 2S_j^z \right) S_i^-.$$

With this definition, the fermion vacuum, $|\{0\}\rangle$ corresponds to the state in which all of the spins point down. The local single particle basis states, defined by $|i\rangle \equiv c_i^\dagger |\{0\}\rangle \equiv |0\rangle^{\otimes i-1} |1\rangle |0\rangle^{\otimes N-i}$, span the single-excitations sector of the total Hilbert space. Restricted to this subspace and expressed in this basis, the Hamiltonian $\hat{\mathcal{H}}$ becomes a tri-diagonal real symmetric matrix \mathbf{T} whose non-zero entries are $t_{nm} = \frac{J_n}{2} (\delta_{n,m+1} + \delta_{n,m-1}) - h_n \delta_{n,m}$. The structure of the eigenvectors of \mathbf{T} and the distribution of its eigenvalues will play a central role in what follows.

As described above, we want to consider the transfer of the entanglement initially shared by spins 0 and 1 into the pair $(0, N)$. To this end, we consider the general state $\rho^{(0,1)}(0)$ at the initial time, and aim at obtaining an expression for the joint state of qubits 0 and N at a subsequent time t .

This latter state, $\rho^{(0N)}(t)$, can be obtained by starting from the unitarily evolved complete density matrix of the full spin system and then tracing out all the degrees of freedom pertaining to spins $1, 2, \dots, N-1$:

$$\rho^{(0N)}(t) = \text{Tr}_{-(0,N)} \left\{ e^{-i\mathcal{H}t} \left[\rho^{(01)}(0) \otimes \Gamma(0) \right] e^{i\mathcal{H}t} \right\}, \quad (3)$$

where $\Gamma(0)$ is the initial density matrix of spins $2, \dots, N$.

In many cases, when investigating the transport properties over a quantum spin channel, one assumes that all of the spins but 0 and 1 are initialized in a fully polarized state; that is, spins sitting at sites 2 to N are prepared in $|0\rangle^{\otimes N-1}$. Because of the $\mathcal{U}(1)$ symmetry of the Hamiltonian, the dynamics is then restricted to the zero- and one-excitation subspaces and Eq. (3) can be straightforwardly expressed in the operator-sum representation¹:

$$\rho^{(0N)}(t) = \sum_{i=0,1} (\mathbf{1} \otimes \mathbf{M}_i(t)) \rho^{(01)}(0) (\mathbf{1} \otimes \mathbf{M}_i(t))^\dagger, \quad (4)$$

where $\mathbf{1}$ is the 2x2 identity matrix acting on the isolated qubit 0, while $\mathbf{M}_i(t)$ are super-operators describing the amplitude damping process of qubit 1:

$$\mathbf{M}_0(t) = \begin{bmatrix} 1 & 0 \\ 0 & u_{N1}(t) \end{bmatrix} ; \quad \mathbf{M}_1(t) = \begin{bmatrix} 0 & \sqrt{1 - |u_{N1}(t)|^2} \\ 0 & 0 \end{bmatrix}.$$

Here the only relevant dynamical parameter entering the Kraus operators is the transition amplitude of the excitation from site 1 to site N , that is, $u_{N1}(t) = \langle N | e^{-i\mathcal{H}t} | 1 \rangle$.

We consider initial input states for the two qubits $(0, 1)$ having an X -type form of the density matrix

$$\rho^{(01)}(0) = \begin{bmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{23}^* & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{bmatrix} \quad \text{with } \sum_{j=1}^4 \rho_{jj} = 1; \quad (5)$$

then, thanks to Eq. (4), the X -type nature is preserved and the only non-zero elements of $\rho^{0N}(t)$ are given by

$$\begin{aligned} \rho_{11}^{(0N)}(t) &= \rho_{11} + (1 - |u_{N1}(t)|^2) \rho_{22}, & \rho_{22}^{(0N)}(t) &= |u_{N1}(t)|^2 \rho_{22}, \\ \rho_{33}^{(0N)}(t) &= \rho_{33} + (1 - |u_{N1}(t)|^2) \rho_{44}, & \rho_{44}^{(0N)}(t) &= |u_{N1}(t)|^2 \rho_{44}, \\ \rho_{14}^{(0N)}(t) &= u_{N1}(t) \rho_{14}, & \rho_{23}^{(0N)}(t) &= u_{N1}(t) \rho_{23}. \end{aligned} \quad (6)$$

The Concurrence C between any two qubit whose density matrix can be expressed in the X -form of Eq. (5), takes a particular simple expression³³: $C = 2 \max[0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}]$. For the case in which the input state $\rho^{(01)}$ is maximally entangled, the Concurrence of the state $\rho^{0N}(t)$ is simply given by $C(\rho^{(0N)}(t)) = |u_{N1}(t)|$.

Several approaches have been put forward in order to maximize this quantity and in the following we will have a closer look at its structure.

The transmission amplitude is given by

$$u_{N1}(t) \equiv \langle N | e^{-i\mathcal{H}t} | 1 \rangle = \sum_{k=1}^N U_{kN} U_{k1} e^{-i\omega_k t}, \quad (7)$$

where U_{ki} is the i -component of the k -eigenvector entering the orthogonal matrix that diagonalizes \mathbf{T} , while ω_k is the corresponding eigenvalue.

Let us first consider the homogeneous channel, $J_i = J = 1$, and $h_i = h \forall i$ (J will be used as the unit for both energy and inverse time). In this case, $U_{ki} = \sqrt{\frac{2}{N+1}} \sin \frac{k\pi}{N+1}$ and $\omega_k = \cos \frac{k\pi}{N+1} - h$, $k = 1, \dots, N$. Notice first that an homogeneous magnetic field contributes only with a global phase to Eq. (7) and, as far as entanglement propagation is concerned, h can be set hereafter to zero without losing generality (this holds true also in the case of non-homogeneous bond constants J_i). Because of the non linear functional form of ω_k and of the many terms entering the sum in Eq. (7), the entanglement propagation is subject to dispersion along the chain, although a finite amount of entanglement between spin 0 and N is always achievable for any finite length N , e.g., for $N = 10^3$ one obtains, at the optimal time t^* , $C(\rho_{0N}(t^*)) \simeq 0.135$. The scaling laws of the attainable entanglement and of the transfer time are given by $C(\rho_{0N}(t^*)) \sim 1.35 N^{-\frac{1}{3}}$ and $t^* \sim N + 0.81 N^{\frac{1}{3}}$, respectively¹.

In the following, we will turn our attention to various proposals aimed at improving the quality of the transfer. A necessary condition³⁴ to be fulfilled by the coupling constants that enter Eq. (1) is that the dynamical matrix \mathbf{T} has to be

mirror-symmetric in order to allow $u_{N1}(t^*)=1$ at some time t^* . This symmetry implies that the eigenvectors of \mathbf{T} are alternatively symmetric and antisymmetric with respect to the center of the chain. For positive (negative) couplings, this yields the condition $U_{k1} = (-1)^k U_{kN}$, where the eigenvectors are ordered in such a way that the corresponding eigenvalues are cast in increasing (decreasing) order. By applying this result to Eq. (7), we obtain a physically more insightful expression:

$$u_{N1}(t) = \sum_{k=1}^N U_{k1}^2 e^{i\phi_k(t)}, \quad (8)$$

which is a sum over all of the eigen-modes of the channel, each evolving with its own phase $\phi_k(t) = (k\pi - \omega_k t)$. Because of dispersion, it is quite unlikely that the phase matching condition $\phi_k(t^*) = \phi_{k'}(t^*)$, which would imply perfect state transmission (PST), will be fulfilled $\forall k, k'$ at some time t^* . In order to allow for PST in a mirror symmetric system, the state evolution has to be periodic, which is equivalent to the requirement that the ratios of the eigenenergy differences have to be rational. It has been shown that, for linear chains with uniform couplings, this prevents PST to occur for $N > 3$ spins⁷. We, then, devote our attention to spin chains that do not preserve translation invariance because either the coupling constants or the magnetic fields are non-uniform.

3.1. Fully-engineered-coupling scheme

One possible strategy adopted in order to satisfy the phase-matching condition is to design the intra-chain couplings in such a way as to realize a linear spectrum, $\omega_k = \alpha k$, with α being a constant. By engineering the couplings according to the rule $J_n = \frac{\pi}{N+1} \sqrt{n(N-n)}$, perfect state transfer can be achieved over arbitrary distances at time $t^* = N+1$ ⁷. Moreover, in Ref.⁹ it has been shown that the requisite of the channel initialization can be relaxed if the coupling are engineered in this way, provided projective measurements are allowed at the end-chain spins.

The determination of specific coupling schemes of a given Hamiltonian in order to achieve the desired quantum correlation transfer process has been addressed also within the Information Flux approach introduced in Ref.³⁵. It has been shown that, if the couplings between neighbor spins can be designed to be non-zero alternatively only along the X - or the Y -axes with specific patterns, then PST is found to occur, together with the generation of maximally entangled states between the end point of the spin chain itself³⁶.

3.2. Entanglement transmission as an effective Rabi oscillation

A completely different strategy to maximize $u_{N1}(t)$ is to let only a few eigen-modes enter the sum in Eq.(8) with a non-negligible weight. In particular, in the ideal case, one would like to have only two contributing terms²³. This can be achieved, for instance, by coupling the end-point spins (those at sites 1 and N) very weakly

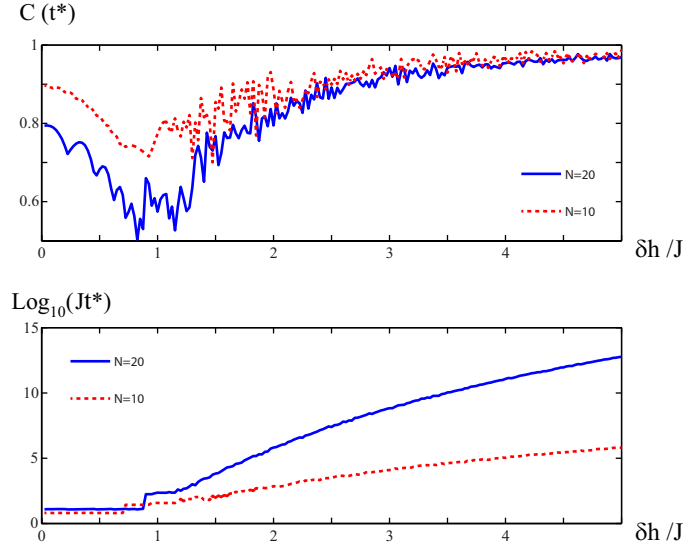


Fig. 2. Transferred entanglement (measured by the concurrence) and optimal transfer times in a spin chain in which the magnetic field is supplemented by two (equal) local fields δh at the sending and receiving sites. In both panels, the solid blue line refer to a chain with $N = 20$ sites, while the dashed red one refers to a chain with $N = 10$ spins.

to their neighbors¹⁹ or by applying there a strong local magnetic field in the z -direction^{21,37} in an otherwise homogeneous system. In these cases, only two eigen-modes become really relevant, corresponding to single excitation states that are bi-localized around the first and the last spins of the chain, which take the approximate form $|\psi_{1,2}\rangle \simeq \frac{1}{\sqrt{2}} (|1\rangle \pm |N\rangle)$.

Rabi-like oscillations, then, take place between these two eigenstates, and an almost perfect entanglement transfer can be achieved at the end of the Rabi period, which is inversely proportional to the difference between the two eigenvalues of the modes involved, $t^* \sim 1/(\omega_2 - \omega_1)$.

The approximate expressions given above become exact in the singular limit in which the couplings become very weak (or the extra, local magnetic field very strong) with respect to the exchange constant of the chain. This, however, occurs at the expense of a much longer transfer time, as in this limit the two states become quasi-degenerate.

This fact can be seen very clearly in the case in which the doublet of states that mainly contributes to Eq. (8) is isolated thanks to an extra local field, δh applied to the first and last spin of the chain. Due to the presence of this extra field, indeed, two quasi-degenerate energy levels are detached in energy from the others and acquire more and more weight in the transition amplitude u_{N1} as δh increases, so that the time evolution that is relevant for the entanglement transport becomes more and more restricted to this bi-dimensional subspace. As a result,

the transferred entanglement substantially increases; but this enhancement of the channel performance comes at the expense of an increased transfer time. Both of the effects are shown in Fig. (2), where the transferred concurrence is shown, together with the time it takes for the transfer to occur.

In the case of weak-end-bonds, on the other hand, the exchange constants connecting spin 1 and 2 and spin $N - 1$ and N , need to satisfy $J_1 = J_{N-1} < J/\sqrt{N}$, so that the transfer time scales linearly with N , see Ref. ¹⁹. This weak-end-bond approach has also been combined with the one with engineered coupling strengths, and it has been shown ³⁸ that, in this case, the performance of the transfer protocol has an enhanced resilience to the presence of random static disorder of the couplings $J_i^{rnd} = J_i(1 + R)$, where R is a uniformly distributed random variable.

3.3. Improving the transmission by optimizing the end-bonds

The two strategies discussed in the previous sections are in a certain sense complementary with respect to the aim of maximizing $u_{N1}(t)$: the first one acts on the phases $\phi_k(t)$ by fine-tuning all of the couplings of the chain in order to have appropriate eigenvalues spacing; whereas, the second one acts on the weights U_{k1}^2 by markedly weakening the effective couplings of spins 1 and N with the rest of the chain (either by acting directly on the bond or by introducing an energy mismatch) in order to obtain only two relevant terms among the ones that enter the sum in Eq. (8).

The question of whether there exists an optimal tradeoff between the linearization of the spectrum and the density of the excited modes, with minimal requirements from the engineerization point of view, has been addressed in Refs. ^{12,39}. There, it has been shown that the end-point coupling strengths influence (and, to some extent, control) at the same time both the eigenenergy spacing and the width of the excited mode density and that there exists a finite optimal value $J_1 < J$ of that coupling, which allows for a very efficient entanglement transfer. In particular, the eigenenergies $\omega_{k_n} = \cos k_n$ experience a shift (with respect to the values they take with uniform couplings) towards the center of the spectrum at $k_0 = \frac{\pi}{2}$ according to the relation

$$k_n = \frac{n\pi + 2\phi_{k_n}}{N + 1}, \quad (9)$$

where $\phi_{k_n} = k_n - \cot^{-1}(\frac{\cot k_n}{\delta})$ and $n=1, \dots, N$. Because of the lowering of the coupling, also the density of the excited modes concentrates towards the center of the spectrum according to

$$U_{k1}^2 = \frac{1}{N + 1 - 2\phi'_{k_n}} \frac{\delta(1 + \delta)}{\delta^2 + \cot^2 k_n}, \quad (10)$$

where $\delta = \frac{J_1^2}{2 - J_1^2}$ (and $J = 1$). The first (and last) bond coupling strength, thus, can be used as a knob for two purposes: i) optimizing the spacing of the frequencies

ω_{k_n} , in order to extend the size of the linear zone around the inflection point $k_0 = \frac{\pi}{2}$, and ii) concentrating more excited modes inside this region.

The existence of an optimal value of J_1 that maximizes $u_{N1}(t)$ is due to the interplay between two conflicting effects: the “linearization” effect, that becomes less pronounced as J_1 is lowered, and the “concentration” effect that, on the contrary, is found to occur more and more as J_1 is lowered.

Once the end-point couplings are set to their optimal values, an almost coherent dynamics arises, yielding an high quality transfer of entanglement in a time $t^* \simeq N + 1.89N^{\frac{1}{3}}$, where the increased delay with respect to the perfectly ballistic transfer time N is precisely due to the fact that the first and last “steps” take more time as $J_1 < 1$. The transferred entanglement can achieve very high values, *e.g.*, $C(\rho_{0N}(t^*)) \simeq 0.89$ for $N = 10^3$. Remarkably, also in the limit $N \rightarrow \infty$, entanglement transfer takes places with an efficiency around 85% and the optimal coupling scales as $J_1^{opt} \simeq N^{-\frac{1}{6}}$, whereas for the all-uniform-coupling case, $C(\rho_{0N}(t^*)) \rightarrow 0$.

These considerations are summarized in Fig. 3, where the (normalized) density of excited modes $D(k) \equiv \frac{U_{k1}^2}{\max_k [U_{k1}^2]}$ and the eigenenergies ω_k entering Eq. (8) are reported for a chain of $N = 100$ spins. The case of uniform couplings is compared with the one in which the couplings are fully optimized, and with that in which only weak end-bonds are included. It is evident that with uniform couplings, the distribution of modes is quite broad and the dispersion relation is manifestly non linear, which yields a poor quality entanglement transfer across long chains. On the contrary, in a chain with fully engineered couplings, the single particle spectrum is completely linear and the density of mode is gaussian, which guarantees a perfect transfer. In the case of a chain with weak end-couplings, there are two quasi-degenerate dominant modes entering the dynamics, which implies that a good transfer quality requires very long times. Finally, the optimal coupling scheme with (slightly) weaker end bonds accomplishes both a shrinking of the relevant modes and an approximate linearization of their dispersion relation

The effect of static disorder on the couplings within the transfer channel, has been also taken into account. The presence of a random contribution in the exchange constants, $J_i^{rnd} = J_i + \delta J_i$, $i = 2, \dots, N - 2$, has been extensively investigated in ^{40,41} for different kinds of static disorder δJ_i where a detailed comparison of the transfer performances of the fully-engineered chain, of the chain with weak-ends and of that with optimal couplings is presented.

A further relevant improvement of the optimal coupling transfer scheme has been put forward in Ref. ⁴² by allowing also the second- (and last-but-one) coupling J_2 (equal to J_{N-2}) to be set at an optimal value, with $J_1 < J_2 < J$. With this additional optimization, the two competing effects of the deformation of the eigenvalue spacing and of the shrinking of the mode distribution can be handled independently, instead of being both controlled by J_1 as in the original scheme. It turns out that the eigenvalues are given by the same expression of Eq. (9), but with the phase shifts ϕ_{k_n} that are mainly ruled by J_2 , whereas the mode density U_{k1}^2 is mainly controlled

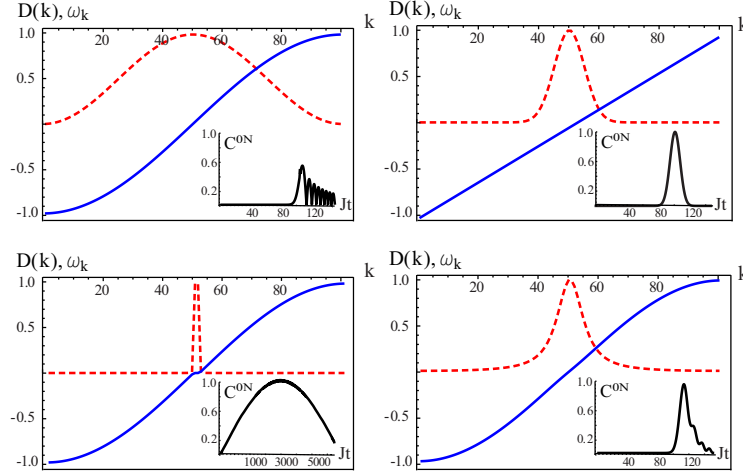


Fig. 3. First row: normalized density of excited modes $D(k) \equiv \frac{U_{k1}^2}{\max_k [U_{k1}^2]}$ (red dashed line) and eigenvalues ω_k , $k=1, \dots, N$ (blue solid line) in the case of the uniform (left plot), and the fully engineered chains (right plot). Second row: same quantities for the weak-end-bond case, with $J_1=10^{-2}$ (left plot) and for optimal couplings $J_1=0.49$ (right plot). The insets show the time evolution of the concurrence between spin 0 and N in the corresponding regimes.

by J_1 . As a result, a considerably higher amount of entanglement can be transferred across very long chains, in a time that scales as $t^* \simeq N + 3.24N^{\frac{1}{3}}$. In the limit $N \rightarrow \infty$, entanglement transfer takes places with an efficiency of more than 99% provided that J_1 and J_2 are set to their optimal values, which scale with the size of the chain as $J_1^{opt} \simeq N^{-\frac{1}{3}}$ and $J_2^{opt} \simeq N^{-\frac{1}{6}}$.

3.4. Propagation of quantum discord

While most of the work in the context of spin-channels has focused on the study of the propagation of entanglement in such media, it is now widely accepted that nonclassical correlations do not reduce to just quantum entanglement. Quantum discord⁴³ and measurement-induced disturbance⁴⁴, just to mention two well known examples, are able to quantify the nonclassical correlation content of a given quantum state well beyond entanglement. Although their role in quantum information processing has not been fully clarified yet, the interest in their properties has constantly increased in the last few years as they are understood as general indicators of the quantumness of a state.

In Ref.²⁹, the authors have addressed the question of whether or not the fundamentally conceptual difference between entanglement \mathcal{E} and discord \mathcal{D} leaves signatures in the way such non-classical quantities are transferred across a spin channel, with the same setting described above, Fig. (1). The question makes full sense both for a pure and a mixed initial state $\rho^{(01)}$, as in both cases the output state of the transfer protocol needs not be pure, so that, in general, $\mathcal{D} \neq \mathcal{E}$. To an-

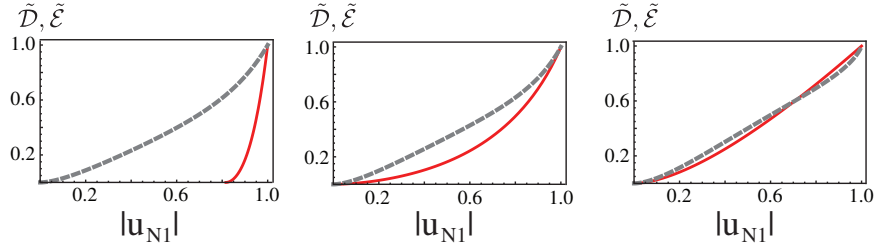


Fig. 4. Comparison between the re-scaled discord $\tilde{\mathcal{D}}$ (grey dashed line) and entanglement of formation $\tilde{\mathcal{E}}$ (red solid one) propagated across a chain of given transition amplitude $|u_{N1}|$. The spin-pair $(1, 0)$ is prepared in a Werner state with $a = 0.4, 0.7$ and 1 [panel (a), (b) and (c), respectively].

analyze the transfer efficiency, the best choice is to use as figure of merit the re-scaled quantities $\tilde{\mathcal{D}} = \mathcal{D}/\mathcal{D}_{(01)}$ and $\tilde{\mathcal{E}} = \mathcal{E}/\mathcal{E}_{(01)}$, where \mathcal{D} is the discord contained in the state $\rho^{(0N)}(t^*)$, while \mathcal{E} is the entanglement of formation of the same state (which is chosen here in place of the concurrence in order to have two entropic-like quantities to compare), and their values are renormalized with respect to the amount of the corresponding quantity in the initial state.

As possible input states of the qubit pair $(0, 1)$, pure and X -type mixed states of the form reported in Eq. (5) have been considered, the latter being of particular relevance because both maximally entangled mixed states and maximally discorded mixed states fall into this class^{45,46}. Despite the substantial differences among the various input states, a common feature of the transport process is that the discord propagates better than entanglement for a wide range of input states and working conditions of the channel; and that $\tilde{\mathcal{E}}$ exceeds $\tilde{\mathcal{D}}$ only when both the transition amplitude and the purity of the initial state are large enough. A representative example of this behavior can be given by choosing $\rho^{(01)}(0)$ to be a Werner state⁴⁷, whose non-null matrix elements in Eq. (5) are given by $\rho_{11} = \rho_{44} = \frac{1+a}{4}$, $\rho_{22} = \rho_{33} = \frac{1-a}{4}$, $\rho_{14} = \frac{a}{2}$, where $-1/3 \leq a \leq 1$ and which is entangled for $a \geq 1/3$. As shown in Fig. (4), for values mildly larger than this threshold, where the purity of the state is small and so is its entanglement, very large values of $|u_{N1}|$ are required in order to actually transport entanglement. On the other hand, the discord \mathcal{D} is non-null for any value of $|u_{N1}|$ and irrespectively of the initial discord content of the pair $(0, 1)$. The relative discrepancy between the two figures of merit is in general very large and decreases only for almost ideal transport conditions. As $a \rightarrow 1$, that is, by increasing the purity of the state, more entanglement is present at the beginning and larger and larger fractions of it are transported, even for small transition amplitudes $|u_{N1}|$. Thus, for increasingly pure initial states, the differences between the two non-classicality indicators are more and more reduced for a good channel. In the limit of $a=1$, which makes $\rho^{(01)}$ a maximally entangled pure state, discord is overtaken by the entanglement of formation at $|u| \geq 1/\sqrt{2}$.

Moreover, for the case of a spin chain with uniform couplings, we have seen that the entanglement of formation of $\rho^{(0N)}(t)$ is invariant with respect to the presence of an homogeneous magnetic field; whereas, for the quantum discord, this is not the case if the initial state $\rho^{(01)}(0)$ displays different values of the spin-spin correlations on the xy -plane; that is, if $\langle\sigma_0^x\sigma_1^x\rangle\neq\langle\sigma_0^y\sigma_1^y\rangle$. In fact, the magnetic field increases the amount of discord that can be obtained between 0 and N as compared to the case with $h=0$.

The formal structure of Eq. (4) can be employed also for the study of the storage of the quantum correlations shared by spin 0 and 1 in the presence of an environment acting only on qubit 1. Thus, the same description presented up to now can be adopted to discuss the preservation of entanglement and of quantum discord in the case in which one of the initially correlated members is acted upon by an environment, constituted in our case by the spins at sites $2, \dots, N$. Since entanglement cannot increase by LOCC, at variance with Discord, strikingly different dynamical behaviors of these two kind of quantum correlations take place in open systems. In fact in Refs. 49,50 it is shown that a local and memoryless environment can indeed generate quantum discord starting from a purely classical state. Moreover, under the effect of Markovian environments, the two quantities can display very different behaviors as entanglement can experience sudden death, while quantum discord can vanish only asymptotically⁵¹ or even stay constant for a finite time interval⁵². These different dynamical behaviors are reproduced in the case of the effective amplitude damping channel obtained by considering our spin chain²⁹.

4. Anisotropic models

In the protocols described so far, the initial state of the quantum channel, (that is, the state of the spins residing at sites ranging from 2 to N) has been chosen to be initialized in the zero-particle sector. As a result, the presence of an additional spin-spin interaction along the z -direction in the Hamiltonian \mathcal{H} , that is introduced in Eq. (1) by letting $\Delta\neq 0$, does not affect significantly the transfer process. This is a consequence of the fact that the dynamical effect of the additional coupling term is equivalent to that of an overall uniform magnetic field, applied to every spin but for the ones at end-points; which implies that the dynamics still remains restricted to the zero- and single-excitation sectors of the total Hilbert space.

On the other hand, if one considers a spin chain that is not fully polarized at the beginning, but that, rather, is prepared, e.g., in its ground state, then the transfer of quantum correlations will strongly depend on the value of Δ ⁴⁸. In particular, the best working point (as far as both the amount of the transferred entanglement and the transfer speed are concerned) turns out to be the $\Delta=1$ anti-ferromagnetic point. An interesting question is whether different channel initializations can result in an improvement of the transfer efficiency. This topic has been addressed in Ref. 32, where a comprehensive analysis of the transport of entanglement has been performed for the anti-ferromagnetic ($J > 0$) XY model (with $\gamma\neq 0$, $\Delta=0$) and for

the XXZ model ($\gamma=0$, $\Delta \neq 0$), by considering different channel's initial states: the ferromagnetic polarized state, in which all of the spins are parallel to each other; the Néel state, in which neighboring spins are antiparallel; and the ground state. As shown in Fig. 5 for a chain made up of $N = 20$ spins, the entanglement turns out to be better transmitted in the XX case ($\gamma=0$), whereas, in the XY model, the transfer capability of the model falls to zero already for moderate values of the anisotropy γ . Furthermore, for the XX model, among the considered initial states, the ferromagnetic one is found to be the most efficient. Nevertheless, the presence of a uniform magnetic field h in the Z -direction enables to obtain a finite amount of entanglement between spin 0 and N also in the presence of strong anisotropy and it turns out that the ground state becomes the most efficient initial state when the field is present.

In the case of the XXZ channel, which, unlike the models with $\Delta=0$, doesn't map into a free fermion system, one has to take into account also the scattering effects between the interacting excitations which gives rise to a more complex behavior of the transmission efficiency, which depends on the phase of the model: different initial states exhibit a strong dependence of the transfer quality for different Δ 's, as reported in Fig. 6 (see Ref. ³² for more details).

5. Concluding remarks

From the point of view of coherent information processing, the nontrivial dispersion properties of spin networks represent an interesting opportunity for their use as short-distance communication channels for the interconnections among on-chip

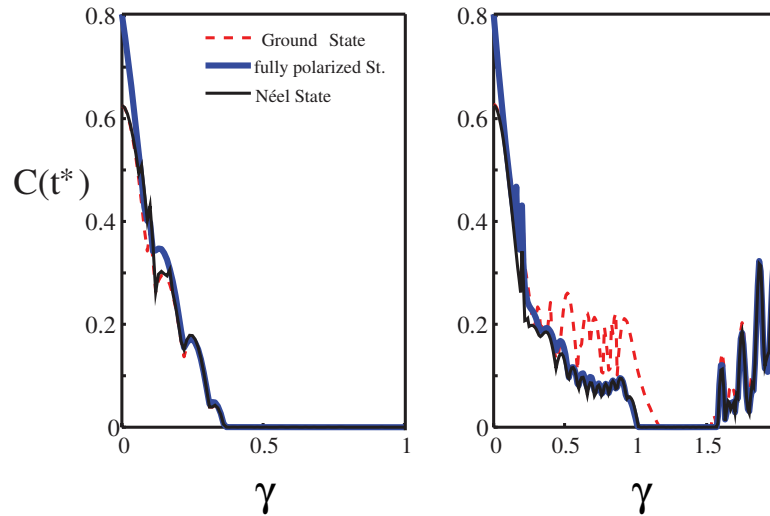


Fig. 5. Concurrence vs in-plane anisotropy γ , for $h=0$ (left) and $h=0.5$ (right) in a chain of length $N=20$ for different initial states: ground state (red-dashed line), ferromagnetic state (blue, thick line) and Néel state (black, thin line).

nodes in the next generation of information processing devices. In this context, in this paper we have reviewed various properties of finite one dimensional spin systems, employable as quantum channels to transmit quantum correlations (and, in particular, entanglement) from one end to the other. The efficiency with which such correlations are propagated across the spin chain appears to be larger for a system with XX interactions between spins initially prepared in the fully polarized state. In such a case, the transfer properties only depend on the modulus of the transfer function $u_{N1}(t)$, given in Eq. (8), that describes the propagation of a localized single-particle (fermion) excitation, and whose value can be adjusted to some extent by employing the coupling constants between neighboring spins as control parameters. In particular, a high quality transmission in an almost ballistic time can be obtained with minimal requirements in terms of the degree of engineering of the chain, if one adopts the optimized end-bond coupling scheme, in which the first and the last bonds are weakened (with respect to all of the others) in such a way as to approximately linearize the excitation spectrum of the modes that effectively contribute to the propagation.

To finish this summary, we just mention that the models that have been considered can be experimentally realized in various ways, ranging from NMR samples to atoms loaded into one-dimensional optical lattices that are able to simulate spin Hamiltonians with a very high degree of accuracy⁵⁴.

6. Acknowledgement

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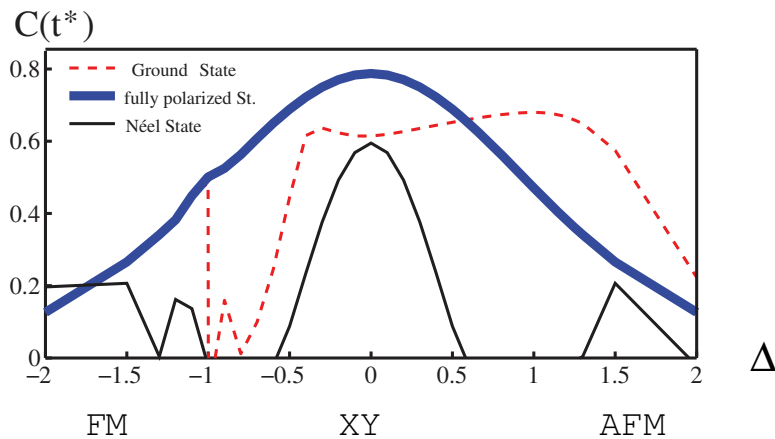


Fig. 6. Attainable Concurrence vs Δ over the complete phase diagram of the XXZ model for different initial states.

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